**Quadratic Equations Lesson #2:**  
**Factoring Trinomials of the Form** \( ax^2 + bx + c \)

The technique for factoring \( ax^2 + bx + c \) by decomposition may have been learned in an earlier course. If this is the case, the next two pages and assignment questions #1-#3 may be omitted. Class Ex. #3 to #5 may be used as practice.

### Review

Complete the following:

a) \( (2x + 1)(3x + 4) = 6x^2 + 11x + 4 \) so \( 6x^2 + 11x + 4 \) factors to \( (2x + 1)(3x + 4) \).

b) \( (3x - 2)(4x + 3) = 12x^2 + x - 6 \) so \( 12x^2 + x - 6 \) factors to \( (3x - 2)(4x + 3) \).

Consider the following problem:  
What are the factors of \( 2x^2 + 7x + 6 \)?

We need to find two binomials whose product is \( 2x^2 + 7x + 6 \).

The first method we will consider is to use algebra tiles.

### Factoring \( ax^2 + bx + c \) Using Algebra Tiles

- **a)** Write a polynomial expression for the group of algebra tiles shown.

  ![Algebra Tiles](image)

  \( 2x^2 + 7x + 6 \)

- **b)** Arrange the algebra tiles into a rectangle and state the length and width of the rectangle.

  ![Rectangle](image)

  Length: \( 2x + 3 \)  
  Width: \( x + 2 \)

  \( (2x + 3)(x + 2) \)

- **c)** Use the algebra tile diagram to express the polynomial in factored form.

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Complete Assignment Questions #1 - #2

Factoring using algebra tiles will work for all trinomials of the form $ax^2 + bx + c$ which have binomial factors. However, it can get rather tedious if the values of $a, b, c$ are large.

**Factoring $ax^2 + bx + c$ Using the Method of Decomposition**

1. In Warm-Up a) we factored
   
   $6x^2 + 3x + 8x + 4$ or $6x^2 + 11x + 4$ to get $(2x + 1)(3x + 4)$.
   
   In order to factor $6x^2 + 11x + 4$, we must first split $11x$ into $3x$ and $8x$ and then group.
   
   But how do we know to split $11x$ into $3x$ and $8x$ rather than $2x$ and $9x$ or $5x$ and $6x$ etc.? In $6x^2 + 11x + 4$, how are the numbers 8 and 3 connected to the value of $a$ (i.e. 6), the value of $b$ (i.e. 11) and the value of $c$ (i.e. 4)?

2. In Warm-Up b) we factored
   
   $12x^2 + 9x - 8x - 6$ or $12x^2 + x - 6$ to get $(3x - 2)(4x + 3)$.
   
   In order to factor $12x^2 + x - 6$, we must first split $1x$ into $9x$ and $-8x$ and then group.
   
   But how do we know to split $1x$ into $9x$ and $-8x$ rather than $5x$ and $-4x$ or $3x$ and $-2x$? In $12x^2 + x - 6$, how are the numbers 9 and $-8$ connected to the value of $a$ (i.e. 12), the value of $b$ (i.e. 1) and the value of $c$ (i.e. $-6$)?

The method of factoring $ax^2 + bx + c$ by splitting the value of $b$ into two integers whose product is $ac$ and whose sum is $b$ is called the **method of decomposition**.

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Factor, using the method of decomposition, and compare the answers with Class Examples #1 and #2.

\[ 2x^2 + 7x + 6 \]
\[ 2 \times (x+2) + 3 \times (x+3) \]
\[ (x+2)(2x+3) \]

\[ 5x^2 + 7x + 2 \]
\[ 5 \times (x+1) + 2 \times (x+1) \]
\[ (x+1)(5x+2) \]

Class Ex. 04

\[ 6x^2 + 17x - 3 \]
\[ 6x(x+3) - 1(x+3) \]
\[ (x+3)(6x-1) \]
\[ -6 \cdot 4 \]

\[ 3x^2 - 2x - 8 \]
\[ -2(3x-4) \]
\[ -2(3x-4) \]

\[ 12t^2 - 8t + 1 \]
\[ 12t^2 - 4t - 4t + 1 \]
\[ 4t(3t-1) - 4t(3t-1) \]
\[ (2t-1)(6t-1) \]

Class Ex. 05

\[ 15 - 7x - 2x^2 \]
\[ -2x^2 - 7x + 15 \]
\[ -2x^2 - 10x + 3x + 15 \]
\[ -2x(x+5) + 3(x+5) \]
\[ (x+5)(-2x+3) \]
\[ 5x \]

\[ 15x^2 + 5x - 10 \]
\[ 5(3x^2 + x - 2) \]
\[ 5(3x^2 + 3x - 2x - 2) \]
\[ 5(3x(x+1) - 2(x+1)) \]
\[ 5(x+1)(3x-2) \]

Complete Assignment Questions #3 - #10

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Assignment

1. a) Write a polynomial expression for the group of algebra tiles shown.

b) Arrange the algebra tiles into a rectangle and state the length and width of the rectangle.

c) Use the algebra tile diagram to express the polynomial in factored form.

2. Factor the following expressions using algebra tiles.
   a) \(2x^2 + 5x + 3\)  
   b) \(2x^2 + 7x + 3\)  
   c) \(6x^2 + 7x + 2\)  
   d) \(4x^2 + 13x + 3\)
3. Factor the following expressions.
   a) \(10x^2 + 17x + 3\)  
   b) \(9x^2 + 6x + 1\)
   
   c) \(3x^2 + 14x + 15\)  
   d) \(3t^2 - 23t - 8\)
   
   e) \(3r^2 + r - 2\)  
   f) \(2x^2 - 19x + 9\)

4. Factor.
   a) \(3x^2 - 2x - 1\)  
   b) \(8y^2 + 2y - 3\)
   
   c) \(9r^2 - 24r + 16\)  
   d) \(12m^2 - 11m - 5\)
   
   e) \(12p^2 + 13p - 4\)  
   f) \(9x^2 - x - 10\)

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5. A rectangular garden has an area of $12x^2 - 5x - 2 \text{ m}^2$.
   a) Write the area of the garden as the product of two binomials with integer coefficients.
   b) Explain how to find the area of the path.
   c) The garden is to be completely enclosed by a path 1 m wide. Find and simplify an expression for the area of the path.
   d) The path is concrete, poured to a depth of 10 cm. Calculate the volume (in m$^3$) of concrete used if $x = 6$.

6. Factor the following expressions.
   a) $12 + 8x + x^2$
   b) $6 - 7x - 20x^2$
   c) $3 + a - 10a^2$
   d) $10a^2 + 25a - 15$

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7. Consider the following, in which each letter represents a whole number.

\[4x^2 + 23x + 15 = (Dx + W)(x + R)\]
\[12x^2 - 52x - 9 = (Ex - T)(Ox + 1)\]
\[16x^2 + 40x - 56 = (x - G)(Ex + S)\]

Determine the value of each letter, and hence name the sports celebrity represented by the following code.

(9) (8) (1) (2) (5) (3) (6) (6) (4) (7)

8. One factor of \(20x^2 + 6x - 8\) is

A. \(2x - 1\)
B. \(4x + 2\)
C. \(5x - 4\)
D. \(10x - 1\)

9. The factored form of \(3x^2 - 14x + 8\) is \((x + a)(bx + c)\) where \(a\), \(b\), and \(c\) are integers. The value of \(2b - a\) is _____.

(Record your answer in the numerical response box from left to right.)
10. The expression $12x^2 - 7x - 10$ can be written in the form $(ax - b)(cx + d)$ where $a, b, c,$ and $d$ are all positive integers.

Write the value of $a$ in the first box. Write the value of $b$ in the second box. Write the value of $c$ in the third box. Write the value of $d$ in the fourth box.

(Record your answer in the numerical response box from left to right.)

Answer Key

1. a) $3x^2 + 7x + 2$  b) $3x + 1, x + 2$  c) $3x^2 + 7x + 2 = (3x + 1)(x + 2)$
2. a) $(2x + 3)(x + 1)$  b) $(2x + 1)(x + 3)$  c) $(3x + 2)(2x + 1)$  d) $(4x + 1)(x + 3)$
3. a) $(5x + 1)(2x + 3)$  b) $(3x + 1)^2$  c) $(3x + 5)(x + 3)$  d) $(3r + 1)(r - 8)$  e) $(3r - 2)(r + 1)$  f) $(2x - 1)(x - 9)$
4. a) $(3x + 1)(x - 1)$  b) $(2y - 1)(4y + 3)$  c) $(3t - 4)^2$  d) $(3m + 1)(4m - 5)$  e) $(4p - 1)(3p + 4)$  f) $(9x - 10)(x + 1)$
5. a) $(3x - 2)(4x + 1)$
   b) Subtract the area of the small rectangle from the area of the large rectangle.
   c) $14x + 2$ m$^2$  d) $8.6$ m$^2$
6. a) $(6 + x)(2 + x)$  b) $(3 + 4x)(2 - 5x)$  c) $(3 - 5a)(1 + 2a)$  d) $5(2a - 1)(a + 3)$
7. TIGER WOODS
8. A
9. 
10. 

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