Physics 12

Non-Uniform Electrical Potential Energy WS

18 marks

1. A 25 μC charge is placed 4.3 m away from a 10 μC charge. What is the potential energy for these two charges?

\[ E_p = \frac{kQq}{r} = \frac{9.00 \times 10^9 \times 2.3 \times 10^{-6} \times 3.45 \times 10^{-6}}{4.3} \]

\[ E_p = 16.63 \text{ J} \]

2. Two equal charges contain a total of 3.5 J of potential energy when they are 5.6 m apart. What is the charge of one of the particles?

\[ E_p = \frac{kQ^2}{r} = \frac{1}{2} kQ^2 \quad \text{(since } Q = Q) \]

\[ Q^2 = \frac{E_p r}{k} \]

\[ Q = \sqrt{\frac{E_p r}{k}} = \sqrt{\frac{3.5 \times 5.6}{9.00 \times 10^9}} \]

\[ Q = 4.66 \times 10^{-5} \text{ C} = 47 \mu\text{C} \]
3. A 34 μC charge is initially placed 2.6 m away from a 78 μC charge. The 34 μC charge is then moved to a position of 1.2 m away from the 78 μC charge as shown.

\[ \text{2.6 m} \quad \text{1.2 m} \quad \text{78 μC} \]

34 μC

a. What is the work done in moving the 34 μC charge? (4 marks)

\[ W = \Delta E_p = E_p_2 - E_p_1 \]

\[ W = \frac{kQ_1Q_2}{r_2^2} - \frac{kQ_1Q_2}{r_1^2} \]

\[ \Delta E_p = \frac{9.00 \times 10^9 \times 31 \times 10^6 \times 78 \times 10^{-6}}{1.2} - \frac{9.00 \times 10^9 \times 31 \times 10^6 \times 78 \times 10^{-6}}{2.6} \]

\[ = 19.89 \text{ J} - 9.18 \text{ J} = +10.71 \text{ J} \] (gains \( E_p \! \! \) !)

b. How much force is required to make the 34 μC move as indicated above? (2 marks)

\[ W = F \times d \]

\[ F = \frac{W}{d} = \frac{10.71 \text{ J}}{1.4 \text{ m}} = 7.64 \text{ N} \]

4. An alpha particle (4 times the mass of a proton and twice its charge) is travelling at 2.4 \( \times \) \( 10^7 \) m/s when it is 8.0 m away from a 7.0 \( \times \) \( 10^{-6} \) C charge. What is the alpha particle's distance of closest approach? (4 marks)

\[ \text{6.67} \times 10^{-11} \text{ kg} \times (2.4 \times 10^7)^2 + 9.80 \times 10^9 \times 3.2 \times 10^{-19} = 7.6 \times 10^{-6} \]

\[ \Rightarrow \frac{9.80 \times 10^9}{r^2} \Rightarrow r^2 = \frac{4.6 \times 10^{-19}}{0.32 \times 10^{-19}} \]

\[ r = 2.0 \text{ m} \]

5. A proton is initially held at rest 1.3 m away from a 65 μC charge. The proton is then released and begins to accelerate away from the 65 μC charge.

\[ 7.8 \text{ m} \quad 1.3 \text{ m} \]

a. What speed will the proton have at a distance of 7.8 m away from the 65 μC charge? (5 marks)

\[ E_{k_1} + E_{p_1} = E_{p_2} \]

\[ \frac{1}{2}m v_1^2 + \frac{kQ_1Q_2}{r_1} = \frac{kQ_1Q_2}{r_2} \]

\[ \frac{1}{2} \times 6.67 \times 10^{-11} \times 1.6 \times 10^{-19} \times 2.4^2 + 9.80 \times 10^9 \times 6.5 \times 10^{-19} = 7.8 \times 10^{-6} \]

\[ \Rightarrow \frac{9.80 \times 10^9}{r_2^2} \Rightarrow r_2^2 = \frac{4.6 \times 10^{-19}}{0.32 \times 10^{-19}} \]

\[ r_2 = 2.0 \text{ m} \]

\[ E_{k_2} = \frac{9.00 \times 10^9 \times 1.6 \times 10^{-19} \times 6.5 \times 10^{-6}}{1.3} = 7.2 \times 10^{-14} \]

\[ E_{p_1} = \frac{kQ_1Q_2}{r_1} = \frac{9.00 \times 10^9 \times 1.6 \times 10^{-19} \times 6.5 \times 10^{-6}}{1.3} = 7.8 \times 10^{-14} \]

\[ F_1 = \frac{kQ_1Q_2}{r_1^2} = \frac{9.00 \times 10^9 \times 1.6 \times 10^{-19} \times 6.5 \times 10^{-6}}{1.3} = 5.0 \times 10^{-14} \]
4. An alpha particle (4 x mass of a proton and twice its charge) is travelling at 2.4 x 10^7 m/s when it is 8.0 m away from a -7.6 x 10^{-16} C charge. What is the alpha particle's distance of closest approach? (4 marks)

\[ \frac{1}{2} m u^2 + k \frac{Q a}{R} = \frac{1}{2} m u^2 + \frac{k Q a}{R} \]

\[ 2.0 \times 10^{-17} \times 9 (4.4 \times 10^8)^2 + \frac{9.0 \times 10^9 \times 3.2 \times 10^{-19}}{2.0} = \frac{k Q a}{R} \]

\[ 4.65 \times 10^{-7} = \frac{k Q a}{R} \]

\[ R = \frac{9.0 \times 10^9 \times 3.2 \times 10^{-19}}{4.65 \times 10^{-7}} \]

5. A proton is initially held at rest 1.3 m away from a 65 \( \mu \)C charge. The proton is then released and begins to accelerate away from the 65 \( \mu \)C charge.

\[ \text{= 7 m} \]

a. What speed will the proton have at a distance of 7.8 m away from the 65 \( \mu \)C charge? (5 marks)

\[ \frac{1}{2} m v^2 + k \frac{Q a}{R} = \frac{1}{2} m v^2 + \frac{k Q a}{R} \]

\[ \frac{1}{2} \times 9.0 \times 10^9 \times 1.6 \times 10^{-19} \times 65 \times 10^{-6} = \frac{9.0 \times 10^9 \times 1.6 \times 10^{-19} \times 65 \times 10^{-6}}{7.8} \]

\[ k \frac{Q a}{R} = 7.2 \times 10^{-14} - 1.2 \times 10^{-14} = 5.0 \times 10^{-14} \]

\[ \frac{1}{2} m v^2 = E_k \]

\[ \therefore u = \sqrt{\frac{2 E_k}{m}} = \sqrt{\frac{2 \times 5.0 \times 10^{-14}}{1.67 \times 10^{-27} \text{kg}}} \]

\[ u = 8.4 \times 10^6 \text{ m/s} \]

b. An electron replaces the proton and the 65 \( \mu \)C is replaced with a -65 \( \mu \)C charge. Explain why the speed of the electron is much greater than the proton even though they both start with the same potential energy. (2 marks)

Since the mass of an electron is much less than a proton (\( m_e = 9.11 \times 10^{-31} \text{ kg} \), \( m_p = 1.61 \times 10^{-27} \text{ kg} \)), the electron would be travelling at a greater speed (for the same \( \Delta E \)) than the proton.
\[ \frac{1}{2} mv^2 = \frac{e^2}{r} \quad \therefore \quad v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{1.67 \times 10^{-27} \text{kg}}{1.6 \times 10^{-19} \text{C}}} \]

\[ v = 8.48 \times 10^6 \text{ m/s} \]

b. An electron replaces the proton and the 65 μC is replaced with a -65 μC charge. Explain why the speed of the electron is much greater than the proton even though they both start with the same potential energy (2 marks)

Since the mass of an electron is much less than a proton (\( m_e = 9.11 \times 10^{-31} \text{ kg} \) vs. \( m_p = 1.6 \times 10^{-27} \text{ kg} \)), the electron would be travelling at a greater speed \( v \) for the same \( \Delta E \) than the proton.