So far we have learned two formulas for principle and interest. These are:

Simple interest \( i = Prt \)  
Compound interest \( M = P(1+i)^n \)

We discovered that simple interest is not used a lot in banking and loans, but rather the interest is compounded. This brings us to another dilemma. In the above formula, the interest is compounded only once per year. This is a problem if money is added to or taken away from the principle (other than interest additions). Bank accounts have their balances regularly changed, and many other situations arise that cause fluctuations in the principle value. So, the solution is to compound \textbf{periodically}, meaning more than once per year on a regular basis. The frequency that the principle (+ interest) is compounded periodically depends on the institution. There are 6 different varieties. See the table below.

<table>
<thead>
<tr>
<th>Name of compounding</th>
<th>Frequency of compounding</th>
<th># of compounds a year (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>annually</td>
<td>once per year</td>
<td>1</td>
</tr>
<tr>
<td>semi-annually</td>
<td>once every 6 months</td>
<td>2</td>
</tr>
<tr>
<td>quarterly</td>
<td>once every 3 months</td>
<td>4</td>
</tr>
<tr>
<td>monthly</td>
<td>once every month</td>
<td>12</td>
</tr>
<tr>
<td>weekly</td>
<td>once every week</td>
<td>52</td>
</tr>
<tr>
<td>daily</td>
<td>once every day</td>
<td>365</td>
</tr>
</tbody>
</table>

Every time the principle is compounded, the interest gets tied back into the principle for the next calculation. If money is added to or removed from the principle, it is reflected in the next calculation. The more frequent the calculation, the more accurately the interest payments reflect the value of the account. Consequently, we have a new formula. It is:

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]

where \( P = \text{principle} \)  
\( r = \text{interest rate (as a decimal)} \)  
\( n = \# \text{ of compounds/year (see table)} \)  
\( t = \text{time in years} \)

Example: Determine the total owed and interest earned on a savings account with a principle of $5000 with an interest rate of 2\( \frac{3}{4} \)% compounded monthly over an 8 year period.

Step 1: calculate \( r \) and \( n \).  
\( r = 2\% \div 100 = 0.02375 \)  
\( n = 12 \) (monthly)

Step 2: plug the values in.  
\[
A = P \left(1 + \frac{r}{n}\right)^{nt} = 5000 \left(1 + \frac{0.02375}{12}\right)^{12 \times 8}
\]

Step 3: solve – this is important – keep all the decimals, do entirely in calculator if possible.

\[
A = 5000 \left(1 + 0.0019791667\right)^{96} \quad A = 5000 \times 1.0019791667^{96} \quad A = 5000 \times 1.2090225
\]

\[
A = 6045.11 \quad i = A - P = 6045.11 - 5000 = 1045.11
\]

\textbf{Activity 1: } Determine the final value of and the earned interest of a $12,000 savings account where the interest rate is 1\( \frac{1}{2} \)% compounded quarterly over 10 years.
Homework:
1. Copy the formula into the box to the far right.
   \[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]
2. Fill in the blanks.
   a) What does the P represent? _______________
   b) What does the letter r represent? _______________ It is expressed as a _____________
   c) What does the letter t represent? _______________
   d) What does the letter n represent? _______________
   e) What is the value of n if the interest is compounded …
      i) annually: ___                  ii) semi-annually: ___                  iii) quarterly: ___
      iv) monthly: ___                v) weekly: ___                        vi) daily: ______
3. What is the formula used to calculate …
   a) Simple interest? _______  b) Compound interest (once annually)? _______
   c) Compound interest (two or more times annually? _____________
4. Determine the value of A (use the periodic compounding formula):
   a) An $8000 loan at 5% compounded quarterly over a 3 year period.
   b) A $11,500 loan at 4½% compounded semi-annually over a 5 year period.
   c) A $35,000 loan at 7¼% compounded monthly over a 6 year period.
   d) A $50,000 loan at 3¾% compounded weekly over a 10 year period.
5. Determine: the loan with interest (A), the interest by itself (i), and the monthly payments.
   a) A $16,000 loan at 4% compounded monthly over a 5 year period.
   b) A $25,000 loan at 8% compounded daily over a 7 year period.