Physics 12

**Gravitation Review Worksheet**

1. Cosmologists are finding new planets that orbit far away stars yearly. A newly discovered planet orbits its own sun at a distance of \(3.0 \times 10^{12}\) m and has a period of \(2.7 \times 10^{10}\) s.
   a. Determine the Kepler’s constant for this system.
      \[
      K = \frac{R^3}{T^2} = \frac{(3.0 \times 10^{12})^3}{(2.7 \times 10^{10})^2} = 3.7 \times 10^{16} \text{ m}^3/\text{s}^2
      \]
   b. Another planet about this same sun orbits with a period of \(5.6 \times 10^9\) s. What is the average orbital radius for this planet?
      \[
      R = \sqrt[3]{KT^2} = \sqrt[3]{3.7 \times 10^{16} \times (5.6 \times 10^9)^2}
      \]
      \[R = 1.1 \times 10^{12}\text{ m}\]
   c. Explain why Kepler’s constant is not a true constant.
      \[K \text{ is different for every different orbiting system } \Rightarrow K_{\text{earth}} \ll K_{\text{sun}}\]

2. The orbiting Space Station orbits the Earth at 250000 m above the Earth’s surface. Use the moon’s period of \(2.36 \times 10^5\) s and orbital radius of \(3.84 \times 10^8\) m to find the period of the Space Station. (How many sunrises do the astronauts on board see every earth day?)
   \[
   \frac{R_m^3}{T_m^2} = \frac{R_s^3}{T_s^2}
   \]
   \[
   \Rightarrow T_s = \sqrt[3]{\frac{R_s^3 \times T_m^2}{R_m^3}} = \sqrt[3]{\frac{(6.38 \times 10^6 \pm 250000)^3 \times (2.36 \times 10^5)^2}{(3.84 \times 10^8)^3}}
   \]
   \[
   \approx 5354 \text{ s } = 5\text{ hours}
   \]

# sunrises:
1 day \(\Rightarrow \frac{86400}{5354} = 16 \times\)
3. A 670 kg robotic spaceship is sitting on the surface of Venus which has a mass of $4.88 \times 10^{24}$ kg and an average radius of $6.07 \times 10^5$ m.
   
   a. Calculate the gravitation force acting on the robotic spaceship while on the surface of Venus.
   
   $$ F = \frac{GmM}{R^2} = \frac{6.67 \times 10^{-11} \times 4.88 \times 10^{24} \times 670}{(6.07 \times 10^5)^2} $$
   
   $$ F = 5900 \text{ N} $$
   
   b. Use this information to determine the gravitational field strength on the surface of Venus.
   
   $$ g = \frac{Gm}{R^2} \quad \text{or} \quad g = \frac{F}{m} $$
   
   $$ g = \frac{6.67 \times 10^{-11} \times 4.88 \times 10^{24}}{(6.07 \times 10^5)^2} = \frac{5919 \text{ N}}{670 \text{ kg}} = 8.8 \frac{\text{N}}{\text{kg}} $$
   
   c. High above the planet Venus is the Robot's Mother ship, orbiting the planet with a period of $1.5 \times 10^4$ s. What is the orbital radius of the Mother ship's orbital radius?
   
   $$ F_c = mg $$
   
   $$ \frac{GMmR^2}{T^2} = \frac{GMm}{R^2} \left( \frac{3\sqrt{\frac{GMm}{R^2}}}{4\pi} \right) $$
   
   $$ R = \sqrt[3]{\frac{GMm}{\frac{3\sqrt{\frac{GMm}{R^2}}}{4\pi}}} $$
   
   $$ R = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 4.88 \times 10^{24} \times (1.9 \times 10^9)^2}{4\pi}} $$
   
   $$ R = 1.4 \times 10^7 \text{ m} $$
4. A newly discovered planet has a radius of $4.5 \times 10^6$ m and a gravitational field strength on its surface of 6.2 m/s². What is the mass of this planet?

$$g = \frac{Gm}{R^2} \quad \therefore m = \frac{gR^2}{G} = \frac{6.2 \times (4.5 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$m = 1.9 \times 10^{24} \text{ kg}$$

5. Mars has a moon called Demos which orbits at an average radius of $2.3 \times 10^7$ m and an orbital period of $1.1 \times 10^5$ s. Use this information to determine the mass of Mars.

$$\frac{GM_m}{R^2} = \frac{4\pi^2R^3}{GT^2}$$

$$M_{Mars} = \frac{\frac{4\pi^2R^3}{GT^2}}{G} = \frac{\frac{4\pi^2 \times (2.3 \times 10^7)^3}{6.67 \times 10^{-11} \times (1.1 \times 10^5)^2}}{6.4 \times 10^{23} \text{ kg}}$$

6. A communication satellite having a mass of $2.1 \times 10^4$ kg orbits the Earth with a period of 24 h (called geo-stationary orbit).

a. Determine the orbital radius for this satellite.

$$R = \sqrt[3]{\frac{GM_T}{\frac{4\pi^2}{T^2}}} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^2 \times (24 \times 3600)^2}{\frac{4\pi^2}{(4.2 \times 10^7)}}}$$

$$R = 4.2 \times 10^7 \text{ m}$$

b. Calculate the gravitational potential energy relative to zero at infinity for this satellite.

$$E_p = -\frac{Gmm}{R} = -\frac{6.67 \times 10^{-11} \times 5.98 \times 10^2 \times 2.1 \times 10^4}{4.2 \times 10^7}$$

$$E_p = -1.49 \times 10^{11} \text{ J}$$
c. Calculate the orbital velocity for this satellite?

\[ \frac{m v^2}{R} = \frac{G m M}{R^2} \]
\[ \therefore \ v = \sqrt{\frac{G m M}{R}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4.2 \times 10^7}} \]
\[ v = 3100 \text{ m/s} \]

d. Determine the escape velocity of this satellite?

\[ \frac{1}{2} m v^2 = \frac{G m M}{R} \]
\[ \therefore \ v_{\text{escape}} = \sqrt{\frac{2 G m M}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4.2 \times 10^7}} \]
\[ v_{\text{escape}} = 4400 \text{ m/s} \]

7. Calculate the gravitational potential energy of the Earth about the Sun at zero relative to infinity.

\[ E_p = -\frac{G m M}{R} \]
\[ = -\frac{6.67 \times 10^{-11} \times 2.0 \times 10^{30} \times 5.98 \times 10^{24}}{1.5 \times 10^{11}} \]
\[ = -5.3 \times 10^{33} \text{ J} \]

That's a lot of energy! The Earth isn't going anywhere other than around our Sun!
8. A rocket is able to lift a 240 kg payload to a vertical height of 120000 m above the Earth's surface.
   a. What is the gain in gravitational potential energy of payload in this situation?

   \[ \Delta E_p = E_{p2} - E_{p1} = \frac{-GMm}{R_2} - \frac{-GMm}{R_1} \]

   \[ \Delta E_p = \left( \frac{-6.67 \times 10^{-11} \times 5.78 \times 10^{24} \times 240}{6.5 \times 10^6} \right) - \left( \frac{-6.67 \times 10^{-11} \times 5.78 \times 10^{24} \times 240}{6.38 \times 10^6} \right) \]

   \[ \Delta E_p = +2.8 \times 10^8 \text{ J} \]

   b. The payload is allowed to crash land back on the Earth's surface. Ignoring air resistance, what is the impact velocity of the payload?

   \[ E_k = \Delta E_p \]

   \[ \frac{1}{2}mv^2 = \Delta E_p \]

   \[ v = \sqrt{\frac{2 \Delta E_p}{m}} \]

   \[ v = \sqrt{\frac{2 \times 2.8 \times 10^8}{240}} \]

   \[ v = 1520 \text{ m/s} \]
9. Calculate the escape velocity from the surface of the Earth and of our Moon. Explain why future space ventures would be better launched from our Moon.

\[ \nu_E = \sqrt{\frac{2GM}{R}} \]
\[ = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^2}{6.38 \times 10^6}} \]
\[ = 11200 \text{ m/s} \]

\[ \nu_m = \sqrt{\frac{2GM}{R}} \]
\[ = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^2}{1.74 \times 10^6}} \]
\[ = 2400 \text{ m/s} \]

\[ \nu_E > \nu_m \]
\[ \therefore \text{we less energy to lift off & escape from the moon!} \]

10. A 720 kg communication satellite is placed in a geostationary orbit. Calculate the total energy of this satellite.

\[ E_T = E_K + E_P \]
\[ E_T = \frac{1}{2}mv^2 + \frac{-GMm}{R} \]
\[ E_T = \frac{1}{2} \frac{mgR}{R} + \frac{-GMm}{R} \]
\[ E_T = \frac{-1}{2} \frac{Gmm}{R} \]
\[ E_T = \frac{-1}{2} \times \frac{6.67 \times 10^{-11} \times 5.98 \times 10^2 \times 720}{4.2 \times 10^7} \]
\[ E_T = -3.4 \times 10^9 \text{ J} \]