Example Problems for Rotational Equilibrium

Rotational Equilibrium is obtained when an object is not rotating and the sum of the torques about a pivot point is equal to zero. We can use this principle to solve some basic problems involving torques. These problems can be categorized as one of the following types:

1] Simple Plank/beam Problems
2] Two-Point Attachment (Bridges) Problems
3] Shelf/Grane Problems
4] Ladder Problems

Let me show you an example of each type:

1] Simple Plank/beam Problems:

A 3.0 m long 7.5 kg beam is balanced by placing a pivot point 1.0 m from one end and having a mass of 67 kg at a distance of 1.5 m from the pivot point as shown.

![Beam Diagram]

What force would be required to keep the beam horizontal?

Solution: Free-body Diagram:

\[
\sum \tau = 0 = \tau_c + \tau_i + \tau_{cc}
\]

\[
\tau = F \times d
\]

\[
0 = F \times 1.0 + (-7.5 \times 9.81 \times 0.50) + (-67 \times 9.81 \times 1.5)
\]

\[
(7.5 \times 9.81 \times 0.50) + (67 \times 9.81 \times 1.5) = F \times 1.0
\]

\[
36.75 \text{ N}\cdot\text{m} + 984.9 \text{ N}\cdot\text{m} = F \times 1.0 \text{ m}
\]

\[
1021.65 \text{ N}\cdot\text{m} = F
\]

\[
1020 \text{ N} = F
\]
2) Bridge Problems:

A 3.5 kg toy truck crosses a 6.7 kg, 1.5 m-long bridge. What are the forces, \( F_L \) and \( F_R \), acting on the bridge from the supports when the truck is 0.35 m from the right end?

(So where is the pivot point? Answer: There are two... one at each end of the bridge!)

Solution: Free body diagram:

Since you have your choice of two pivot points, one at each end, choose one that makes your life easier. In this case place the pivot point on the right side. Force \( F_R \) will not generate a torque because it is acting act the pivot.

\[
\sum \tau = \tau_{CC_1} + \tau_{CC_2}
\]

\[
F_L \times 1.5 = (3.5 \times 9.81 \times 0.35) + (6.7 \times 9.81 \times 0.75)
\]

\[
F_L = \frac{61.25 \, \text{N} \cdot \text{m}}{1.5 \, \text{m}} = 41 \, \text{N}
\]

To solve for \( F_R \) you could just move the pivot point to the left side and re-calculate the torque about that point. However, you have to be careful about determining the new lever arm length!

But there is an easier way!

Remember that the object is not moving from place to place so it must be in translational equilibrium. This means that the sum of forces acting on the object must be equal to zero.

\[
F_{up} = F_{down} \quad (\text{to find } F_R)
\]
\[ F_{up} = F_{down} \quad \text{(to find } F_c) \]

\[ F_L + F_R = F_{gb} + F_t \]

\[ 40.8 \text{ N} + F_R = (6.7 \times 9.81) + (3.5 \times 9.81) \]

\[ F_R = 65.727 + 34.335 - 40.8 \]

\[ F_R = 59 \text{ N} \quad (> F_L) \]

3) Shelf or Crane Problems

These are both done the same way:

- Shelf:
- Crane:

Let's start off with a shelf problem.
A 0.50 m long, 1.2 kg shelf supports a 23 kg mass at its end as shown.

What is the tension in the rope holding the shelf in place?

**Free-Body Diagram:**

\[ \sum \tau = 0 \]
\[ \sum F_y = 0 \]
\[ \sum F_x = 0 \]

1. \[ F_H = 269 N \]
2. \[ F_V = 310 N \]
3. \[ F_x = 12.981 N \]
4. \[ F_y = 23 \times 9.81 N \]
5. \[ F_\perp = 225.63 N \]

\[ \sum F_y = F_V - F_H \]
\[ F_y = \frac{F_\perp}{\cos 65^\circ} \times 0.20 m = 117 N \]
\[ F_\perp = 225.63 N \times 0.50 m = 112.7 N \]
\[ F_\perp = 2.9 N \cdot m + 112.7 N \cdot m \]

**Equilibrium:**
\[ \sum F_y = F_V - F_H \]
\[ F_\perp = 225.63 N \times 0.50 m = 112.7 N \]

\[ F_\perp = \frac{115.64}{0.1813} = 637 N \]

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Now there are other forces acting on the frame, $F_1$ and $F_2$. These were not part of the torque calculation because they act at the pivot point and hence create no torque. But now that we have calculated the tension we can determine these two forces.

Remember: Sum of the Force $= 0$, in order to be in translational equilibrium.

Since there are only two directions here: Sum of the $X$-forces $= 0$ and Sum of $Y$-forces $= 0$.

The only two X-forces are $F_H$ and Tension X-component force. These two must be equal.

$$F_H = F_{T_H} = F_T \times \cos 65^\circ$$
$$F_H = 637 \times \cos 65^\circ = 269 \text{N} \quad \rightarrow \quad \sum F_Y = 0$$

There are 3 Y-forces shown: Tension Y-component, $F_{T_Y}$ and $F_{G_1}$. Let's see if they are in balance.

**Up forces:**

$$F_{T_Y} = F_T \times \sin 65^\circ$$
$$F_{T_Y} = 637 \times \sin 65^\circ = 577 \text{N} \quad \uparrow$$

**Down forces:**

$$F_{G_1} + F_{G_2}$$
$$11.76 \text{N} + 225.4 \text{N} = 237.16 \text{N} \quad \downarrow$$

As you can see you there is more Up force than Down force. The missing force must be acting at the pivot to keep our object in static equilibrium.

Therefore:

$$F_V = F_H - F_{G_2}$$
$$F_V = 577 \text{N} - 237.16 \text{N}$$
$$F_V = 340 \text{N}$$

**Crane Problem:**

76 kg Boom at 6.5 m long

What maximum mass can be supported by this crane?

Free-body Diagram:

$$T_1 = 12000 \text{N} \times \sin 50^\circ = 12000 \times \cos 40^\circ$$
Free-body Diagram:

\[ T_1 = 12000 \times \sin 50^\circ = 12000 \times \cos 40^\circ \]

\[ F_{mg} = F_{g_m} \times \cos 50^\circ \]

\[ F_{g_m} = mg \]

\[ F_{gb} = F_{g_b} \times \cos 50^\circ \]

\[ \tau_{c_1} = \tau_{c_2} + \tau_{cc} \]

\[ \tau_{cc} = \tau_{c_1} + \tau_{c_2} \]

\[ T_1 \times l = F_{g_m \perp} \times l_1 + F_{g_b \perp} \times l_2 \]

\[ 12000 \times \sin 50^\circ \times 5.5 = m \times 9.81 \times \cos 50^\circ \times 6.55 + 76 \times 9.81 \times \cos 50^\circ \times 3.25 \]

\[ 50558.93 N \cdot m = m \times 40.99 + 1557.52 \]

\[ \frac{50558.93 - 1557.52}{40.99} = m \]

\[ 1200 \text{ kg} = m \]
A 62 kg person is 6 ft of the way up a ladder of mass 22 kg. What is the minimum coefficient of friction between the ladder and the floor? (Assume the wall is frictionless.)

The two forces $F_w$ and $F_\ell$ do not contribute to the torque because they act at the pivot point. The force of friction has to be equal to the $F_f$ (see the diagram). You will need to determine the perpendicular forces (see the diagram on the right).

\[ \tau_c = \tau_{c1} + \tau_{c2} \]

\[ F_w \times \sin 72^\circ = 62 \times 9.81 \times \cos 72^\circ \times \frac{3}{4} \times 2 + 22 \times 9.81 \times \cos 72^\circ \times \frac{1}{3} \]

\[ F_w \times 0.9511 = 140.96 + 33.35 \]

\[ F_w = \frac{174.31}{0.9511} = 183 \text{ N} - F_f \]

\[ F_f = \mu F_N \]

\[ \mu = \frac{F_f}{F_N} = \frac{183 \text{ N}}{(62 \times 9.81) + (22 \times 9.81)} \]

\[ \mu = 0.22 \text{ minimum} \]